P425/2	S6 **NUMERICAL METHODS** Time : 1 hour	
1	Use trapezium rule with 5 strips to evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx} dx$, correct to four decimal places.	*05marks*
2	(a) Use the trapezium rule with 6 ordinates to estimate the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} tanx\ \ dx$, correct to 3 significant figures. (b) (i)Calculate the percentage error in your estimation in (a) above . (ii) suggest how the percentage error above may be reduced.	*06marks* *06marks*
3	The table below is an extract from the tables of $cosx^{\circ}$ $x = 80°$ | $0'$ | $10'$ | $20'$ | $30'$ | $40'$ $cosx$ | 0.1736 | 0.1708 | 0.1679 | 0.1650 | 0.1622 Use linear interpolation to estimate (a) $cos\ 80°\ 36'$ (b) $cos^{-1}0.1685$	*05marks*
4	Show that the equation $lnx = 2 - x$ has a root between $x = 1$ and $x = 2$. Hence use linear interpolation only once to find the root correct to 2 decimal places.	*05marks*
5	A cylindrical pipe has a radius of 2.5cm measured to the nearest unit. If the absolute relative error in calculating its volume is 0.125, find the absolute relative error in measuring its height;	*05marks*
6	(a) Given that Δx and Δy are the errors made in approximating x and y to given numbers of decimal places respectively respectively. Show that the maximum absolute error in $\frac{y}{x^2}$ is $2\left\|\frac{y\Delta x}{x^3}\right\| + \left\|\frac{\Delta y}{x^2}\right\|$. (b) hence find the (i) limits within which the exact value of $\frac{0.74}{1.6^2}$ (ii) percentage error made in $\frac{0.74}{1.6^2}$. give your answers to 3sf	*12marks*

P425/2 **S6 NUMERICAL METHODS**

TEST 2 MARKING GUIDE 2020

Qn		marks
1.	$h = \frac{\frac{\pi}{2}-0}{5} = \frac{\pi}{10}$	B1 for h

x	Extreme values	Middle values
0	1.00000	
$\frac{\pi}{10}$		0.76393
$\frac{2\pi}{10}$		0.62981
$\frac{3\pi}{10}$		0.55279
$\frac{4\pi}{10}$		0.51254
$\frac{\pi}{2}$	0.50000	
totals	1.50000	2.45907

B1 *for all values of x to5dp*

B1 *for all values of yto5dp*

$$\therefore \int_0^2 \frac{1}{1+sinx}dx \approx \frac{1}{2}\times\frac{\pi}{10}[1.5+(2\times 2.45907)]$$

M1 *iff ≈ seen*

$$\approx 1.00816$$
$$\approx 1.008$$

A1 *cao 3dps*

05

Qn		marks
2.	$h = \frac{\frac{\pi}{3}-\frac{\pi}{6}}{5} = \frac{\pi}{30}$	**B1** for h

x	Extreme values	Middle values
$\frac{\pi}{6}$	0.5774	
$\frac{\pi}{5}$		0.7265
$\frac{7\pi}{30}$		0.9004
$\frac{4\pi}{15}$		1.1106
$\frac{3\pi}{10}$		1.3764
$\frac{\pi}{3}$	1.7321	
totals	2.3095	4.1139

B1 *for all values of x to 4dp*

B1 *for all values of y to 4dp*

$$\therefore \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} tanx\ dx \approx \frac{1}{2}\times\frac{\pi}{30}[2.3095+(2\times 4.1139)]$$

M1 *iff ≈ seen*

$$\approx 0.5517 \approx 0.552 \text{ (3sf)}$$

A1 *cao 3sf*

Exact value = $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} tanx\ dx = [-\ln cosx]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

M1 *for integrating &substituting the limits*

$$= -\ln cos\tfrac{\pi}{3} - -\ln cos\tfrac{\pi}{6}$$
$$= 0.5493 \approx 0.549\ (3sf)$$

B1 *cao to 3sf*

$$percentage\ error = \left|\frac{exact\ value - approximate\ value}{exact\ value}\right| \times 100$$

M1 *iff modulus sign included*

$$= \left|\frac{0.549-0.552}{0.549}\right| \times 100$$

B1

$$= 0.5464\%$$

A1

	(ii) the percentage error can be reduced by increasing the number of ordinates .	**B1**
		12
3.(a)	$x = 80°$ | $30'$ | $36'$ | $40'$; $cosx$ | 0.1650 | y_1 | 0.1622	**B1** *for the table*
	$\frac{0.1622-0.1650}{40'-30'} = \frac{y_1 - 0.1650}{36'-30'}$	**M1** *for the gradient application*
	$\therefore y_1 = 0.1633$	**A1** cao to 4dp
(b)	$x = 80°$ | $10'$ | x_1 | $20'$; $cosx$ | 0.1708 | 0.1685 | 0.1679	
	$\frac{0.1679-0.1708}{20'-10'} = \frac{0.1685-0.1708}{x_1 - 10'}$	**M1** *for the gradient application*
	$x_1 = 18'$ $\therefore cos^{-1}0.1685 = 80°18'$	**A1** cao *take note the answer is not 18'* but $80°18'$
		05
4.	Let $f(x) = \ln x + x - 2$	**B1 for f(x)**
	$f(1) = \ln 1 + 1 - 2 = -1$ $f(2) = \ln 2 + 2 - 2 = 0.6931$	**B1** for subs. For x=1 & x=2
	Since $f(1) < 0$ and $f(2) > 0$ then there exists a root between $x = 1$ and $x = 2$.	**B1** *for comment*
	x | 1 | x_1 | 2 ; $f(x)$ | -1 | 0 | 0.6931	
	$\frac{0.6931--1}{2-1} = \frac{0--1}{x_1 - 1}$	**M1** *for the gradient application*
	$x_1 = 1.5906$ ≈ 1.59	**A1** cao
		05
5.	$v = \pi r^2 h$	
	$\Delta v = \frac{\delta v}{\delta r}\Delta r + \frac{\delta v}{\delta h}\Delta h$	**B1for partial differentiation**
	$\Delta v = 2\pi rh\Delta r + \pi r^2 \Delta h$ $\left\|\frac{\Delta v}{v}\right\| = \left\|\frac{2\pi rh\Delta r + \pi r^2\Delta h}{\pi r^2 h}\right\|$	**B1 for relative error**
	$\left\|\frac{\Delta v}{v}\right\| = \left\|\frac{2\Delta r}{r} + \frac{\Delta h}{h}\right\|$ $\leq \left\|\frac{2\Delta r}{r}\right\| + \left\|\frac{\Delta h}{h}\right\|$	**M1 for triangular inequality**
	$\left\|\frac{\Delta v}{v}\right\|_{max} = 2\left\|\frac{\Delta r}{r}\right\| + \left\|\frac{\Delta h}{h}\right\|$	**B1**

	$$0.125 = 2\left\|\frac{0.05}{2.5}\right\| + \left\|\frac{\Delta h}{h}\right\|$$ $$\left\|\frac{\Delta h}{h}\right\| = 0.085$$	**A1**
		05
6.	Let X and Y be the exact values of x and y respectively such that $X = x + \Delta x$, and $Y = y + \Delta y$ Error = exact value – approximate value $= \frac{y+\Delta y}{(x+\Delta x)^2} - \frac{y}{x^2}$ $$= \frac{(y+\Delta y)(x)^2 - y(x+\Delta x)^2}{x^2(x+\Delta x)^2}$$ $$= \frac{= x^2y + x^2\Delta y - y(x^2 + 2x\Delta x + (\Delta x)^2)}{x^2(x+\Delta x)^2}$$ $$= \frac{= x^2y + x^2\Delta y - x^2y - 2xy\Delta x - (\Delta x)^2 y}{\left(x^2\left(1+\frac{\Delta x}{x}\right)\right)^2}$$	**M1**
	Assuming $\Delta x <<< x, then\ (\Delta x)^2 y \approx 0$, and $\frac{\Delta x}{x} \approx 0 \therefore 1 + \frac{\Delta x}{x} \approx 1$ Error $= \frac{x^2\Delta y - 2xy\Delta x}{x^4} = \frac{\Delta y}{x^2} - \frac{2y\Delta x}{x^3}$	**B1 for the assumption**
	Absolute error = $\left\|\frac{\Delta y}{x^2} - \frac{2y\Delta x}{x^3}\right\|$	**M1** *for the modulus sign*
	$$\leq \left\|\frac{\Delta y}{x^2}\right\| + \left\|-\frac{2y\Delta x}{x^3}\right\| \text{ but } \left\|-\frac{2y\Delta x}{x^3}\right\| = \left\|\frac{2y\Delta x}{x^3}\right\|$$ $$\leq \left\|\frac{\Delta y}{x^2}\right\| + \left\|\frac{2y\Delta x}{x^3}\right\|$$ $$\leq \left\|\frac{\Delta y}{x^2}\right\| + 2\left\|\frac{y\Delta x}{x^3}\right\|$$	**B1 for triangular inequality**
	$$\therefore maximum\ absolute\ error = \left\|\frac{\Delta y}{x^2}\right\| + 2\left\|\frac{y\Delta x}{x^3}\right\|$$ (b) for $\frac{0.74}{1.6^2}$, $working\ value = \frac{0.74}{1.6^2} = 0.28906$	**A1** *for the absolute error*
	$$\frac{0.74}{1.6^2} \quad , x = 1.6 , \Delta x = \pm 0.05$$ $$y = 0.74, \Delta y = \pm 0.005$$	**B1 for errors in x and y**

	$maximum\ absolute\ error = \left\|\frac{\Delta y}{x^2}\right\| + 2\left\|\frac{y\Delta x}{x^3}\right\|$ $= \left\|\frac{0.005}{1.6^2}\right\| + 2\left\|\frac{0.74 \times 0.05}{1.6^3}\right\|$ $= 0.02002$ $lower\ limit = working\ value - error$ $= 0.28906 - 0.02002$ $= 0.26904$ $= 0.269$ $upper\ limit = working\ value + error$ $= 0.28906 + 0.02002$ $= 0.30908$ $= 0.309$ ii) $percentage\ error = \left\|\frac{0.02002}{0.28906}\right\| \times 100$ $= 6.9259$ $= 6.93\%$	**M1 for substitution** **A1** **M1** **A1** **A1** **M1** *iff the modulus sign is seen* **A1**